A General Model that Accounts for Fitts’ Law and Drury’s Model

Jui-Feng Lin, Colin G. Drury, Mark H. Karwan, and Victor Paquet

University at Buffalo: SUNY, Department of Industrial and System Engineering
438 Bell Hall, Buffalo, NY 14221, USA

A general model is proposed to account for the rationale of both Fitts’ law (1954) and Drury’s (1971) model to describe the relationships of speed-accuracy tradeoffs (RSATs) in self-paced aiming and self-paced tracking movements, respectively. Based on the concepts of control theory, the general model postulates that to perform hand-control movements, humans behave as an intermittent correction servo associated with four essential motor properties: 1) corrective reaction time, 2) ballistic movement time, 3) ballistic movement variability, and 4) moving behavior and strategy. The general model is specified to the two types of movements: the self-paced aiming movement model and the self-paced tracking movement model. A preliminary test of the two models was conducted against published data. The results showed that both specified models predict well the linear RSATs of Fitts’ law and Drury’s (1971) model. To further validate the model predictions with real movement performance, a comprehensive study was suggested for future research.

INTRODUCTION

Since Woodworth’s (1899) pioneering study, considerable research has been devoted to study the relationship of speed-accuracy tradeoff (RSAT) mainly related to two types of movements: aiming movements (terminal control movements) and tracking movements (path control movements). Aiming movements require individuals to reach a target by moving a determined distance, such as moving a cursor to a desired icon on a computer screen. Tracking movements control an object by following a given path, such as navigating a cursor from the start menu in the Windows® to open Word beneath the Microsoft Office icon.

Fitts’ law (1954) and Drury’s (1971) model are two of the most representative models for predicting the RSATs of self-paced aiming and self-paced tracking movements, respectively. The term, “self-paced”, indicates that movement speed is controlled by participants themselves. Normally, while performing movements, participants are asked to move as fast as they can but maintain a certain movement accuracy. Alternatively, participants may be asked to perform movements at a certain frequency, such as following a metronome. In this case, the movements refer to “paced movements”. In this article, the focus is on self-paced movements. As shown in Equation 1, Fitts’ law shows that the movement time (MT) for an aiming movement is logarithmically related to the ratio of twice the movement amplitude (A) and the target width ($W_{\text{target}}$).

$$MT = a + b \times \log_2(2A/W_{\text{target}})$$  \hspace{1cm} (1)

where $a$ and $b$ are experimentally determined terms. The logarithmic term is called the “index of difficulty” (ID). On the other hand, Drury’s model (Equation 2) shows that movement speed ($V$) of a tracking movement is linearly related to the track’s path width ($W_{\text{path}}$).

$$V = K \times W_{\text{path}}$$  \hspace{1cm} (2)

where $K$ is an experimental constant that represents the controllability. Both models have been validated for numerous types of tasks with promising results [see Lin (2009) for review]. By measuring dependent variables (i.e., MT or $V$) under certain manipulated levels of the relevant independent variables, both models can well describe the RSATs and further predict the unmeasured dependent variables. Beyond the experimental predictions, many researchers were more interested in studying the physiological fundamentals for understanding how and why humans behave in the ways the two models describe.

Although Fitts (1954) originally developed his model with information theory concepts, feedback concepts of control theory are more commonly accepted by subsequent researchers for modeling hand-control movements. The feedback concepts of control theory can be derived from Craik (1947, 1948) and Vince (1947, 1948) who suggested that while performing movements human behaves as an intermittent correction servo that performs a movement by intermittently generating several sub-movements. This idea was further applied in several studies to explain the rationale of Fitts’ law and to model tracking movements.

The studies of Crossman & Goodeve (1963/1983) and Keele’s (1968) together have been accepted as among the best available accounts of Fitts’ law. Their deterministic iterative-corrections model states that movements are made in rapid succession. Each sub-movement is assumed to travel a constant proportion of the distance toward the target. There is a fixed period of time (i.e., corrective reaction time denoted as $t_r$) in which each sub-movement is performed. With these assumptions, their model demonstrates that the total MT is a result of the product of $t_r$ and the number of sub-movements required for completing an aiming movement. The model was further enhanced by Keele (1968) who used an...
experimentally measured $t_r$ of 200 milliseconds and assumed the fixed proportion value of 1/7. Although their models were developed with several doubtful assumptions (e.g., variability of sub-movements, the fixed proportion value), they showed the potential of applying control theory concepts in modeling Fitts’ law.

Another explanation of Fitts’ law was made by Meyer and his colleagues (Meyer, Abrams, Kornblum, Wright, & Smith, 1988; Meyer, Smith, Kornblum, Abrams, & Wright, 1990) who proposed stochastic optimized sub-movements models. With agreement on the intermittent feature, they proposed that an aiming movement was made with two or more sub-movements. However, they disagreed about the deterministic feature and suggested the existence of motor variability. Due to stochastic noise in the human motor system, sub-movements cannot always accurately reach their anticipated end-points. They assumed that the endpoints of a sub-movement formed a normal distribution around the target and could be predicted by the impulse-variability model (Meyer, Smith, & Wright, 1982; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979). Hence, their models were developed by conceptualizing individuals’ strategy for coping with the motor variability of sub-movements to minimize the total MT. Their multiple-sub-movement model (Meyer et al., 1990) predicts well the RSAT of Fitts’ law as the number of sub-movements increases towards infinity. Although some theoretical issues remained in their models (e.g., the impulse-variability model), Meyer and his colleagues’ studies made contributions by involving motor variability in modeling Fitts’ law.

By applying the concepts of the intermittent correction servo, Drury and his colleagues (Drury, Montazer, & Karwan, 1987; Montazer & Drury, 1989) accounted well for the rationale of Drury’s (1971) model. According to their optimal models, a tracking movement also consists of sub-movements made within sampling periods (i.e., corrective reaction time). Associated with each sub-movement, endpoint variability was considered as a bivariate normal distribution and was simply determined by the error measured perpendicular to the movement direction. To cope with the motor variability, their model was thus developed based on the concept that the human operator would choose a specific distance to move in a $t_r$, giving an subjectively acceptable probability of not moving outside the tracked path. Although some issues related to the methods utilized to measure and determine the bivariate normal distribution were reported, Drury and his colleagues made important contributions – it was the first time that individuals’ measured motor variability was utilized to predict their own RSATs of tracking movements.

In summary, by applying feedback concepts of control theory, researchers have tried to account for the rationale behind Fitts’ law and Drury’s (1971) model separately. The RSAT described by Drury’s (1971) model was explained by Drury, et al. (1987). However, the question of how the RSAT described by Fitts law results from the human motor system is still not resolved. Although both aiming movements and tracking movements have been studied individually throughout, the RSATs for both types of movements should result from the same mechanism of the human motor system.

The main purpose of the current research is to propose a general model that represents the physiological fundamentals of our human motor system. With the understandings of these “roots”, the proposed general model is expected to explain how the RSATs of both types of hand-control movements result from the human motor system. This general model is developed by unifying valid concepts, models and further reasonable assumptions. In the following, the proposed general model will first be detailed. We then introduce a self-paced aiming movement model and a self-paced tracking movement model specified based on the general model to predict the two types of RSATs. Finally, the results of preliminary tests of the two specified models with published data are presented.

**PROPOSED MODELS**

### General Model

The general model explains how a human operator performs hand-control movements as an intermittent correction servo associated with four motor properties: 1) correction reaction time, 2) ballistic movement time, 3) ballistic movement variability, and 4) moving behavior and strategy. Valid concepts, relevant models and additional assumptions of the four motor properties will be detailed below.

**Intermittent correction servo.** Along the lines of the previous researchers (e.g., Craik, 1947, 1948; Crossman & Gooden, 1963/1983; Drury et al., 1987; Vince, 1947, 1948), while modeling human motor control, we adopted the feedback concepts of control theory shown in a typical diagram as Figure 1. The terms with parentheses in the figure are normally used to represent a mechanical servo. To utilize this servo as an analogy for explaining how a human operator performs movements, the alternative terms are defined related to the human motor system as shown in this figure. To explain the concepts, consider a hand-control aiming movement in which a human operator is about to move a stylus from a start point to reach an aimed target. The “sensor” (i.e., eyes) first locates the aimed target as “reference”. Information about the target location then is compared to the position of the stylus, currently at the start point, to determine the “error” (i.e., misalignment or movement distance). Once the first movement distance is defined, the brain as “controller” starts to process a movement impulse that is programmed to move the stylus to the aimed target and is called the “system input”. The impulse as an order then tells the “system” (i.e., the controlled limbs and the stylus) to execute the first movement, called the “system output.” Since the movement is executed by a movement impulse under open-loop conditions, it is a “ballistic movement.” Due to the existing noise of the system, variability is associated with the endpoints of the first ballistic movement – the movement might miss the target. Thus, after the execution of the first ballistic movement, the eyes re-determine the remained distance from the target. If any misalignment remains, this closed-loop process iteratively continues to execute further ballistic movements for misalignment corrections until the target is reached, as an “intermittent correction servo”. The performance of this servo is mainly determined by how fast the closed-loop process can be processed and the variability of the ballistic movements. Due to one of the characteristics of the variability – it increases with increased movement distance, to make an accurate movement (i.e., to reach a small target), more ballistic move-
ments are required since the variability of the last ballistic movement needs be small enough to end up within the aimed target, explaining the RSAT while performing aiming movements. The four motor properties of the intermittent correction servo will be detailed below.

**Figure 1. Intermittent correction servo**

Corrective reaction time (tᵣ). This is the period of time required by the servo to generate a ballistic movement for misalignment correction, including receiving visual feedback, programming movement impulses, and sending impulses to the controlled limbs. It was originally named the “psychological refractory period” (Welford, 1952) during which the servo is too busy for generating a new ballistic movement and the ongoing ballistic movement executed after the previous period cannot be autonomously modified. It was found that tᵣ ranges from 190 to 290 milliseconds (Beggs & Howarth, 1970; Drury, 1994; Keele & Posner, 1968).

**Ballistic movement.** A ballistic movement as defined in this article is a movement executed by a single movement impulse. It is executed as a whole and cannot be autonomously modified until it is finished or the next ballistic movement is ready for executing. The distance covered by a ballistic movement was called the “ballistic movement distance” denoted as dᵤ.

**Ballistic movement time (t_ballistic).** The time required for performing a ballistic movement can be shorter than, equal to or longer than tᵣ. It was hypothesized that Gan & Hoffmann’s (1988) model (Equation 3) could be utilized as a ballistic movement time model to predict the relationship between t_ballistic and dᵤ.

\[
t_{\text{ballistic}} = a + b \times \sqrt{d_u}
\]  
(3)

where a and b are constants.

**Ballistic movement variability.** The endpoint variability of a ballistic movement is called the “ballistic movement variability” and has been found to be normally distributed (Crossman & Goodeve, 1963/1983; Fitts, 1954; Fitts & Radford, 1966; Welford, 1968; Woodworth, 1899). When movements are performed on a two-dimensional (2-D) plane, a 2-D endpoint distribution needs to be considered. It was hypothesized that Howarth et al.’s (1971) model (Equation 4) could be utilized as a ballistic movement variability model to predict the relationship between the ballistic movement variability measured both longitudinally (σₓ^2) and perpendicularly (σᵧ^2) to the moving direction. The dichotomy of the two types of variability recognizes the fact that the amplitude of longitudinal variability is larger than that of lateral variability (Beggs et al., 1972; Beggs, Sakstein, & Howarth, 1974; Murata & Iwase, 1999; Schmidt et al., 1979). The way to determine the 2-D endpoint distribution is illustrated in Figure 2 in which the aimed location vicinity of a ballistic movement (the dash arrow) is divided into small squares. To obtain the 2-D distribution is thus equivalent to calculating the probabilities that the endpoints would be located in all the individual squares.

The probability of each square, P(Sᵢ,j), can be simply determined by multiplying the associated lateral probability and longitudinal probability that are determined by the relevant lateral normal distribution (Lateral Xᵢ) and the relevant longitudinal normal distribution (Longitudinal Xⱼ). According to the general model, these two types of distributions can be predicted by the ballistic movement variability model (Equation 4). The lateral distributions were assumed to be calculated according to actual ballistic movement distances (dᵤ). Hence, the amplitude of Lateral Xᵢ is larger than that of Lateral Xᵢ=1, since dᵤ of Lateral Xᵢ=1 is longer than that of Lateral Xᵢ=1. However, the longitudinal distributions are assumed to be the same for every row of squares and calculated according to dᵤ. Therefore, the amplitudes of Longitudinal Xⱼ=0 and Longitudinal Xⱼ=15 are the same.

\[\sigma^2 = a + b \times d_u^2\]

(4)

where a and b are constants and \(\sigma\) is the standard deviation of the endpoint normal distribution.

**Moving behavior and strategy.** Moving behavior refers to the description of how a tracking movement is composed of ballistic movements. And moving strategy is referred to as the way in which dᵤ is chosen by the human operator. Unlike the other three motor properties that keep consistent across different types of movements, different types of movements require different moving behaviors to finish the movements and need different strategies to obtain better performance. Therefore, moving behavior and strategy need to be specified according to certain types of movements.

In the following two sections, the details of the four motor properties, especially moving behavior and strategy, will be specified to self-paced aiming and self-paced tracking movements, formatting two specified models for predicting the RSATs.

**Specified Self-Paced Aiming Movement Model**

In addition to the general model defined above, the self-paced aiming movement model was developed further based on the proposed moving behavior and strategy shown in
The expected MT can be obtained by multiplying every possible combination of ballistic movements (n\textsubscript{ballistic}) required for completing the aiming movement. In the case shown in Figure 3, three regions are defined as an example. If the first ballistic movement’s endpoints are inside the target [Region 1 in Figure 3 (a)], the movement ends with the first ballistic movement. If the endpoints are in Region 2 shown in Figure 3 (b), two ballistic movements are required to finish the movement since all of the endpoints of the second ballistic movements are inside the target. And if the endpoints are in Region 3 as shown in Figure 3 (c), the movement needs either two or three ballistic movements to be completed. Based on this simplified concept, the expected MT can be obtained by multiplying every possible combination of ballistic movements for finishing the aiming movement with their associated probabilities. And, the expected MT can be obtained by taking \( t_{\text{ballistic}} \) and \( t_r \) into account. It was assumed that if a ballistic movement is not the last one to finish the aiming movement and its \( t_{\text{ballistic}} \) is shorter than \( t_r \), there is a “compensatory delay” of \( t_r - t_{\text{ballistic}} \) added to that ballistic movement. Furthermore, one research question asked is whether or not there is a “reaction delay” of \( t_c/2 \) between the first and the second ballistic movements, indicating the average time required to wait for the next available ballistic movement.

Based on the general model, the self-paced tracking movement model was further developed according to the proposed moving behavior and strategy illustrated in Figure 4, using simplified concepts of Drury et al.’s (1987) model but with a sounder way to determine the 2-D endpoint distribution. To perform a tracking movement, it was assumed that the human operator subsequently performs ballistic movements until the tracked path is completed. Each ballistic movement was assumed to be performed within \( t_r \), so that visual feedback can be optimally utilized. Endpoints of each ballistic movement were assumed to be a 2-D distribution that is determined as mentioned in the general model. An example of the calculated bivariate normal distribution is shown as two circular contours in Figure 4. The 2-D distribution and the tracked path width (\( W_{\text{path}} \)) together, thus, determine the probabilities ending outside the path \( [P(Fail)] \) and inside the path \( [P(\text{Success})] \) of that ballistic movement. The self-paced tracking movement model was thus developed by conceptualizing the human operator’s willingness of how far (i.e., \( d_u \)) to move in \( t_r \) to obtain a subjectively optimal (low) probability of ending outside the path \( [P(Fail)] \). If \( P(Fail) \) keeps consistent, \( d_u \) as well as movement speed \( (V) \) increase as \( W_{\text{path}} \) increases and vice versa, explaining the speed-accuracy tradeoff. Furthermore, if monetary reward and penalty are associated with movement performance, the model built based on these concepts will be able to predict the human operator’s RSAT on optimally performing tracking movements.
MODEL SETTINGS

Self-Paced Aiming Movement Model

The self-paced aiming movement proposed above was
programmed in Visual Basic as a simulation to predict MT and \( t_{\text{ballistic}} \) of the self-paced aiming movements. To ob-
tain the model predictions, \( t_r \) were tested with two values of
190 and 290 milliseconds, and the studies by Gan & Hoff-
mann’s (1988) and Beggs, Sakstein & Howarth’s (1974) were
chosen to determine the ballistic movement time model, the
ballistic movement variability model, and the compared expe-

dmental data in the preliminary test.

The ballistic movement time model. This was obtained
from the study by Gan & Hoffmann (1988), in which the
self-paced aiming movements were measured by discretely
tapping a target plate in a left to right direction with six par-
ticipants. The 10 measured ID values, the four amplitudes,
and the measured average movement times are shown in Table
1. In the study, the relevant target widths (\( W_{\text{target}} \)) were
determined according to Equation (1). The MTs measured
with ID values from 1.0 to 3.0 were utilized to validate Equa-
tion 3, in which the movements were considered as ballistic
movements. To eliminate the variability of amplitudes the
participants “actually” moved, only the MTs measured with
ID value equal to 3.0 were utilized to predict ballistic move-
ment time, shown as Equation (5) below, accounting for
93.90 % of the variance.

\[
\begin{align*}
\text{Table 1: Average MTs in millisecond (Gan & Hoffmann, 1988)} \\
\begin{array}{l|cccc}
\text{ID (bit)} & 40 & 90 & 160 & 250 \\
\hline
1.0 & 107 & 137 & 165 & 191 \\
1.5 & 117 & 143 & 162 & 188 \\
2.0 & 118 & 137 & 168 & 191 \\
2.5 & 125 & 150 & 167 & 194 \\
3.0 & 137 & 151 & 169 & 202 \\
3.5 & 151 & 162 & 184 & 208 \\
4.0 & 167 & 178 & 195 & 216 \\
4.5 & 192 & 193 & 213 & 225 \\
5.0 & 216 & 236 & 247 & 248 \\
6.0 & 256 & 285 & 306 & 322 \\
\end{array}
\end{align*}
\]

\[
\tau_{\text{ballistic}} = 90.2 + 21.3 \times \sqrt{d_u} 
\]

The ballistic movement variability model. This was ob-
tained from the study by Beggs, Sakstein & Howarth (1974),
in which ballistic movement variability was measured with 12
participants by discretely tapping a target cross in a left to
right direction. The movements were ballistic since the light
turned off while moving. Four values of \( d_u \) were measured:
200, 300, 400, and 500 mm. Note that, the movements were
paced by a metronome set at 120 ticks/minute no matter how
long the distances. The endpoints variability were measured
parallel and perpendicular to the movement direction. Ac-
cording to the self-paced aiming movement model, only lon-
gitudinal variability was considered to predict the 1-D end-
points distribution. The longitudinal ballistic movement model
is shown in Equation 6 and accounts for 97.55 % of the
variance.

\[
\sigma^2 = 20.87 + 0.00031 \times d_u^2
\]

Compared experimental data. The aiming movements
conducted with the ID values from 3.5 to 6.0 in Gan & Hoff-
mann (1988) were considered as “correct answers” to compare
with the simulation predictions. The average MTs are shown
in Table 1 as well, in which the movements were conducted
while visual feedback were able to benefit the movement ac-
curacy.

Experimental variables. To compare the simulated data
with the data measured by Gan & Hoffmann (1988), the simi-
lar manipulation of ID was utilized. Six ID values of 3.5, 4.0,
4.5, 5.0, 5.5 and 6.0 were manipulated according to the four
amplitudes, 40, 90, 160, and 250 mm.

Self-Paced Tracking Movement Model

The self-paced tracking movement proposed above was
programmed in Visual Basic as a simulation to predict \( V \) of the
tracking movements. Like the self-paced aiming movement
model, two values of 190 and 290 milliseconds were tested as
\( t_r \). However, the way to calculate \( d_u \) did not follow the
optimization method utilized in Drury (1987). Instead, it was
simply determined by the maximum \( d_u \) that could be moved
with zero probability of moving outside the path within \( t_r \).
The three studies by Beggs, Sakstein & Howarth (1974),
Montazer & Drury (1989) and Drury et al. (1987) were chosen
to determine two sets of ballistic movement models and the
compared experimental data.

The ballistic movement variability model. Two sets of data
were utilized to predict the 2-D endpoints distributions. The
first set of data was obtained from Beggs, et al. (1974). In
additional to the longitudinal ballistic movement variability
model (Equation 6), the lateral ballistic movement variability
measured in the same study was utilized to determine to the
2-D endpoints distribution. The lateral ballistic movement
model is shown in Equation 7 and accounts for 98.82 % of the
variance. The other set of data was obtained from Montazer
& Drury (1989), in which only the lateral ballistic movement
variability was measured by asking six participants to draw
lines within four straight line tracks from left to right across
the body, using a ball point pens. The path widths were 15,
20, 30, and 40 mm. As the participants traced a path, the
ambient lighting was turned off to give the last 50, 150, 250
and 350 mm of measured movements in the dark. To obtain
the variability that the participants did their best to minimize,
only the movement performed under the longest dark distance
(i.e., 350 mm) was utilized, representing the lateral ballistic
movement model shown in Equation 8 which accounts for
98.2 % of the variance. It was assumed that Equation 8 also
represented the longitudinal ballistic movement variability
model since it was not available.

\[
\sigma_r^2 = 13.95 + 0.00023 \times d_u^2
\]

\[
\sigma_r^2 = 3.06 + 0.00023 \times d_u^2
\]
ment drew lines along straight tracked paths of length 250 mm. Six path widths, ranged from 1 to 6 mm at 1 mm intervals, were measured. After practice trials on path widths of 2 and 5 mm, the participants drew along each path twice with an experimental design of two Latin Squares. The results of the two trials are shown as the two regression lines (Equation 9 and Equation 10) which accounted for 99.36 % and 97.50 % of the variance, respectively. The other set of data was obtained from the study by Drury et al. (1987), including both the experimental data and model predictions. In their experiment, six participants performed the self-paced tracking movement within straight line tracks (800mm long). Six path widths were 10, 20, 30, 40, 50, and 60 mm and the average speeds obtained by scaling (Fig. 6, Drury et al., 1987) were 0.40, 0.67, 1.00, 1.45, 1.62 and 1.92 (mm/millisecond), respectively. Regression of $V$ on to $W_{path}$ accounted for 98.7 % of the variance. Their optimization model predictions were 0.40, 0.67, 1.00, 1.45, 1.62 and 1.92 (mm/millisecond), respectively. Regression of $MT$ on to ID according to the two

$MT = 0.82 + 6.33/W_{path}$

(9)

$MT = 1.16 + 4.79/W_{path}$

(10)

Experimental variables. To compare the model predictions with the two sets of published data by Beggs et al. (1974) and Drury et al. (1987), the simulation was run twice based on the manipulations of $W_{path}$ as (1) 1, 2, 3, 4, 5 and 6 (mm), and (2) 10, 20, 30, 40, 50 and 60 (mm).

**MODEL PREDICTIONS**

Self-Paced Aiming Movement Model

Movement time ($MT$). The predictions of the self-paced aiming movement model were 24 MTs associated with the experimental combinations. Regressions of the average MT on to ID according to the two $t_r$ values and the two reaction delay conditions (0 and $t_r/2$) are shown in Table 2 and Figure 5 with the regression obtained from Gan & Hoffmann (1988). The simulation model successfully predicted the linear relationship between MT and ID (all $r^2 > 0.95$, $p \leq 0.001$). However, a statistical test on equality of the coefficient of regression lines using the Chow test (1960) showed significant differences between the experimental data and model predictions (all $p$ values <0.001).

Table 2_Comparison of regressions of MT on to ID

<table>
<thead>
<tr>
<th>Data</th>
<th>$t_r$ (ms)</th>
<th>Delay (ms)</th>
<th>Intercept (mm)</th>
<th>Slope (ms/mm)</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>-</td>
<td>-</td>
<td>1.93</td>
<td>47.30</td>
<td>0.965</td>
</tr>
<tr>
<td>Model</td>
<td>190</td>
<td>0</td>
<td>161.8</td>
<td>49.82</td>
<td>0.964</td>
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<td>Model</td>
<td>190 $t_r/2$</td>
<td></td>
<td>94.94</td>
<td>72.25</td>
<td>0.980</td>
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<tr>
<td>Model</td>
<td>290</td>
<td>0</td>
<td>120.5</td>
<td>62.57</td>
<td>0.951</td>
</tr>
<tr>
<td>Model</td>
<td>290 $t_r/2$</td>
<td></td>
<td>20.63</td>
<td>93.91</td>
<td>0.973</td>
</tr>
</tbody>
</table>

Figure 5_MT/ID relationships of Gan & Hoffmann and the model

Number of Ballistic Movements ($n_{ballistic}$). The other dependent variable predicted by the self-paced aiming movement model was $n_{ballistic}$ that was not affected by the changes of the $t_r$ values and the reaction delay conditions. Regression of $n_{ballistic}$ on to ID is shown in Figure 6 and accounts for 97.0 % of the variance.

Figure 6_Number of ballistic movement/ID relationship of the model

Self-Paced Tracking Movement Model

Comparison with Beggs et al. (1974). The predictions of the self-paced tracking movement model were six speed values associated with the manipulated path widths. Regressions of $V$ on to $W_{path}$ according to the two $t_r$ values are shown in Table 3 and Figure 7 with the experimental data calculated from Equation 9 and Equation 10. The simulation model successfully predicted the linear relationship between $V$ and $W_{path}$ (all $r^2 = 1.0$, $p < 0.001$). However, a statistical test on equality of the coefficient of regression lines using the Chow test showed (1) highly significant differences between the experimental data and model prediction with $t_r = 290$ milliseconds ($p \leq 0.001$) and (2) significant differences between the experimental data and model prediction with $t_r = 190$ ($p = 0.018$ and 0.011 for experiment 1 and experiment 2, respectively).
Table 3_Comparison of regressions of speed on to width

<table>
<thead>
<tr>
<th>Data</th>
<th>$t_r$ (ms)</th>
<th>Intercept (mm)</th>
<th>Slope (ms/mm)</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1</td>
<td>-</td>
<td>0.0219</td>
<td>0.1950</td>
<td>-</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>-</td>
<td>0.0338</td>
<td>0.0168</td>
<td>-</td>
</tr>
<tr>
<td>Model</td>
<td>190</td>
<td>-0.0026</td>
<td>0.0263</td>
<td>1.0</td>
</tr>
<tr>
<td>Model</td>
<td>290</td>
<td>-0.0017</td>
<td>0.0172</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 7_Speed/Width relationships of Beggs et al. and the model

Comparison with Drury et al. (1987). Regressions of $V$ on to $W_{path}$ according to the two $t_r$ values are shown in Table 4 and Figure 8 with the experimental data and model predictions obtained from Drury et al. (1987). The simulation model successfully predicted the linear relationship between $V$ and $W_{path}$ ($r^2 = 0.972, p < 0.001$). A statistical test on equality of the coefficient of regression lines using the Chow test showed (1) significant differences between both the experimental and model data of Drury et al. (1987) and the model prediction with $t_r = 290$ milliseconds ($p < 0.05$) and (2) no significant differences between both the experimental and model data of Drury et al. (1987) and the model prediction with $t_r = 190$ ($p = 0.214$ and 0.107, respectively).

Table 4_Comparison of regressions of speed on to width

<table>
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<th>Data</th>
<th>$t_r$ (ms)</th>
<th>Intercept (mm)</th>
<th>Slope (ms/mm)</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (Drury et al.)</td>
<td>-</td>
<td>0.0867</td>
<td>0.0311</td>
<td>0.987</td>
</tr>
<tr>
<td>Model (Drury et al.)</td>
<td>290</td>
<td>-0.179</td>
<td>0.0332</td>
<td>-</td>
</tr>
<tr>
<td>Model</td>
<td>190</td>
<td>-0.278</td>
<td>0.0400</td>
<td>0.972</td>
</tr>
<tr>
<td>Model</td>
<td>290</td>
<td>-0.182</td>
<td>0.0262</td>
<td>0.972</td>
</tr>
</tbody>
</table>

DISCUSSION

The preliminary tests of the self-paced aiming movement model and the self-paced tracking movement model showed that both models successfully predicted the linear relationships of speed-accuracy tradeoffs, but that they did not exactly predict the movement performance obtained from the published data. The following discussion will focus on the self-paced aiming movement model and the self-paced tracking movement model. Then, some limitations of this research and suggested future research will be given.

Self-Paced Aiming Movement Model

The proposed self-paced aiming movement model was developed according to the proposed general model that enhances the applications of an intermittent correction servo (Craik, 1947, 1948; Crossman & Goodeve, 1963/1983; Drury et al., 1987; Vince, 1947, 1948). By emphasizing the four motor properties, the self-paced aiming movement model accounts for the relationship of speed-accuracy tradeoff better than the deterministic iterative-corrections model proposed by Crossman & Goodeve (1963/1983) and Keele (1968) and the stochastic optimized sub-movements models by Meyer and his colleagues (Meyer et al., 1988; Meyer et al., 1990). With Crossman & Goodeve (1963/1983), we agree that the human motor system needs at least $t_r$ to generate a ballistic movement. However, we state that the time of a ballistic movement does not need to be equal to $t_r$. Instead, we utilized Gan & Hoffmann’s (1988) model as the ballistic movement time model to predict $t_{ballistic}$. Like Meyer and his colleagues (Meyer et al., 1988; Meyer et al., 1990), we agree with the existing variability associated with a ballistic movement performed under open-loop conditions. However, instead of the impulse-variability model (Schmidt et al., 1979), we state the utilization of Howarth et al.’s (1971) model as the ballistic movement variability model to predict the endpoint distribution of a ballistic movement. Furthermore, like the stochastic optimized sub-movements models, our model assumes that each ballistic movement is aimed at the center of the target width. However, instead of considering $n_{ballistic}$ as a pre-determined variable utilized to obtain movement time (Meyer et al., 1988; Meyer et al., 1990), our model considers $n_{ballistic}$ as a dependent variable that is determined by one of the human operator’s motor properties, ballistic movement variability, and the aiming movement determined by $A$ and $W_{target}$. As shown in Figure 6, the model predicts that $n_{ballistic}$ is also linearly related to $ID$. This relationship is first be found in this research and also in line with many researchers (e.g., Annett, Golby, & Kay, 1958; Carlton, 1980; Jagacinski, Repperger, Ward, & Moran, 1980; Langolf, Chaf-
found one or several sub-movements while performing an aiming movement. The validity of the proposed self-paced aiming movement model could be supported by the preliminary tests of the literature data (Beggs et al., 1974; Gan & Hoffmann, 1988), showing that the model successfully predicted the linear relationships between MT and ID (see Table 2 and Figure 5).

However, the preliminary test of the self-paced movement model also showed that the proposed model did not predict the performance of aiming movements by Gan & Hoffmann’s (1988) participants. As shown in Figure 5, no matter what manipulations of $t_r$ and reaction delay, our model predicted longer MTs than those measured in Gan & Hoffmann’s (1988). We consider that this difference might be mainly due to the reason that the aiming movements performed by their participants were not purely self-paced aiming movements. According to a part of their description, “the subject was allowed practice trials to familiarize him/herself before the start of each condition”, the participants had familiarized the location of the aimed target and the length of the movement amplitude before performing measured movements, benefiting kinesthetic feedback (Carson, Chua, Elliot, & Goodman, 1990; Carson, Goodman, Chua, & Elliot, 1993) and reducing the variability of programming the movement impulses of the first ballistic movement. Therefore, the aiming movements tended to be completed with only the first ballistic movements without further misalignment corrections, resulting in shorter MTs. This hypothesis could be supported by the results shown in their Figure 2 (Gan & Hoffmann, 1988), in which all the MTs were linearly related to $\sqrt{A}$ (all the $r^2 > 0.88$). Hence, the discrepancy of the experimental measurements by Gan & Hoffmann (1988) and the model predictions does not diminish the model validity.

Self-Paced Tracking Movement Model

The self-paced tracking movement model shares similar concepts with the optimization model developed by Drury et al. (1987). However, we refined these concepts in one proposed general model. While the width of tracked path is narrow enough to affect the movement speed of a tracking movement, the model states that the human operator would perform ballistic movements in each period of $t_r$ to complete the self-paced tracking movement. Therefore, $V$ is mainly determined by the length of $t_r$, ballistic movement variability, the probability of movement success and $W_{path}$. We believe that Howarth et al.’s (1971) model can be utilized to predict the 2-D endpoint distribution of a ballistic movement with a longitudinal ballistic movement model and a lateral ballistic movement model. Instead of utilizing the techniques of optimization by Drury et al. (1987), the model simply determine $d_p$ by calculating the maximum length that could be moved without moving outside the track paths; that is, the probability of movement success was set at one. The validity of the self-paced tracking movement model is supported by the preliminary test of the two sets of literature data (Beggs et al., 1974; Drury et al., 1987; Montazer & Drury, 1989), showing that the model successfully predicted the linear relationships between $V$ and $W_{path}$ (see Table 3, Table 4, Figure 7 and Figure 8).

Although the proposed model did not exactly predict the results of self-paced tracking movements measured by (Beggs et al., 1974) and (Drury et al., 1987), the similarity between the experimental measurements and model predictions is much better than that while testing the self-paced aiming movement model. In both sets of tests, our self-paced tracking movement model predicted close results while $t_r$ was set at 190 milliseconds. Specifically, the statistical tests on equality of the regression lines showed the $p$ value was larger than 0.01 for the comparison with Beggs et al. (1974) and was larger than 0.1 for the comparison with Drury et al.’s (1987). The results also showed that our model tended to predict lower $V$ values compared to the experimental measurements. This could be explained by the probability of movement success being set to one, resulting in shorter $d_p$ compared to the experimental measurements in which errors occurred. Therefore, it is more likely that the model is valid for predicting self-paced tracking movements.

Limitations and Future Research

Although the two specified models have successfully predicted the linear relationships predicted by Fitts law (1954) and Drury’s (1971) model with the preliminary tests of published data, some research hypotheses, experimental problems, statistical issues remained in this study. To validate the proposed general model as well as the two specified models, further investigations are required. First of all, $t_r$ was assumed to be in a range from 190 to 290 milliseconds. A direction measurement of individuals’ $t_r$ can help the model validation. Secondly, the application of Gan & Hoffmann’s (1988) model (Equation 3) to predict ballistic movement time needs to be validated under the conditions in which $t_{ballistic}$ is longer than $t_r$. In the original experiment by Gan & Hoffmann (1988), the ballistic movements were only conducted under 200 milliseconds. However, the general model states that the ballistic movement time model can predict $t_{ballistic}$ even if a ballistic movement requires longer than a $t_r$ to be finished. Hence, a direct experimental test is required to validate the model application. Thirdly, the utilization of Howarth et al.’s (1971) model (Equation 4) to predict both the longitudinal variability ($\sigma_L^2$) and the lateral variability ($\sigma_L^2$) needs to be tested as well. Although it was validated in the study by Beggs et al. (1974), the ballistic movements their participants performed were paced by a metronome set at 120 ticks/minute. Also, the amplitudes of the two types of variability in their study seem to be too close. According to other research (e.g., Schmidt et al., 1979), the longitudinal error was much larger than the lateral error. Finally, a sound study that consists of several experiments is required to validate the proposed models. The study should measure the same group of participants with their individual $t_r$, $t_{ballistic}$, $\sigma_L^2$, $\sigma_L^2$, self-paced aiming movements and self-paced tracking movements. The utilization of the data obtained from the literature in this study leaves some issues related to the comparisons between the measured performance and model predictions. For example, as mentioned above the issue in Gan & Hoffmann (1988), their self-paced aiming movements tended to be “ballistic.” Also, the ballistic movement variability utilized to predict the aiming movements were obtained from a different study by Beggs et al. (1974) in which the participants and the experiment were different from those in Gan & Hoffmann...
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REFERENCES


