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Modeling Fitts' law

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ABSTRACT: The model proposed in Lin, Drury, Karwan, & Paquet (2009) was only tested against published data, limiting model validation. The purpose of the current pilot study was to validate the application of the general model for modeling Fitts' law in three designed experiments. Four graduate students participated in the experiments to measure their (1) ballistic movement time and variability, (2) the relationship between movement time and index of difficulty, and (3) the relationship between the number of ballistic movements and index of difficulty. The motor properties measured in the first experiment were utilized by our proposed general model to predict the individuals' relationships measured in the last two experiments. The comparisons of the experimental and the predicted relationships showed that the designed experiments were a feasible basis for further model validation. Some experimental modifications will be required for future research.

Keywords: Fitts' law, intermittent correction servo, aiming movement, goal-directed movement

1 INTRODUCTION

Using "Fitts' law" as keywords, one can easily get more than 13,600 relevant research papers using Google Scholar. The popularity of Fitts' law (1954) is mainly due to its promised results for many different types of movements, manipulations, environments, and participant populations [see (Lin, 2009) for review].

Fitts' law, as shown (Eq. 1), describes the speed-accuracy tradeoff relationship while performing self-paced aiming movements in which a human controls an object to reach a target by moving a certain distance according to his/her own determined speed.

$$MT = a + b \times \log_2 \frac{2A}{W} \qquad (1)$$

where MT is movement time; a and b are experimentally determined constants; the logarithmic term is called "Index of Difficulty (ID)" where A is movement amplitude and W is target width.

Although Fitts' (1954) law was originally developed based on information theory concepts, some researchers consider that the feedback concepts of control theory might explain Fitts' law better. According to Craik, (1947, 1948) and Vince, (1947, 1948), while performing movements the human behaves as an intermittent correction servo that completes a movement by intermittently generating several sub-movements. The concept of intermittent correction servo was further applied in several studies to explain the rationale of Fitts' law.

The studies of Crossman & Goodeve, (1963/1983) and Keele, (1968) together have been accepted as viable accounts of Fitts' law. Their deterministic iterative-corrections model states that movements are made in rapid succession. Each sub-movement is assumed to travel a constant proportion of the distance and to the target in a fixed period of time (i.e., corrective reaction time denoted as t_r). With these assumptions, their model demonstrates that the total MT is a result of the product of t_r and the number of sub-movements required for completing an aiming movement. The model was further enhanced by Keele, (1968) who used an experimentally measured t_r of 200 ms and the assumed fixed proportion value of 1/7. Although the deterministic iterative-corrections model were developed with several doubtful assumptions (e.g., invariability of sub-movements and the fixed proportion value), the model shows the potential of applying control theory concepts in modeling Fitts' law.

Another explanation of Fitts' law was made by Meyer and his colleagues (Meyer, Abrams, Kornblum, Wright & Smith 1988, Meyer, Smith, Kornblum, Abrams & Wright 1990) who proposed stochastic optimized sub-movements models. Meyer and his colleagues also agreed on the intermittent feature and stated that an aiming movement was made with two or more sub-movements. However, they disagreed about the deterministic feature stated by Crossman & Goodeve (1963/1983) and suggested the existence of motor variability. To account for motor variability, they assumed that the endpoints of a sub-movement formed a normal distribution and could be predicted by the impulse-variability model (Meyer, Smith & Wright 1982, Schmidt, Zelaznik, Hawkins, Frank & Quinn, 1979). By conceptualizing individuals' strategy for coping with the motor variability of sub-movements to Minimize the Total *MT*, their multiple-sub-movement model (Meyer et al. 1990) predicts well the speed-accuracy tradeoffs relationships predicted by Fitts' law as the number of sub-movements increases towards infinity. Although Meyer and his colleagues' studies didn't explain how the corrective reaction time plays a role in our motor control system, their studies made contributions by involving motor variability while modeling Fitts' law.

More recently, Lin, Drury, Karwan & Paquet, (2009) proposed a general model that enhanced the concepts of the intermittent correction servo with four specified motor properties: corrective reaction time (t_r), ballistic movement time, ballistic movement variability, and moving behavior and strategy. In the general model, the sub-movement mentioned above was defined as the "ballistic movement" that is executed by a single movement impulse. Once it is executed, it cannot be autonomously modified until it is completed or the next ballistic movement is ready for executing. Similar to the concepts used by Crossman & Goodeve (1963/1983), the length of t_r would affect the execution of ballistic movement. However, the time required for performing a ballistic movement, called the "ballistic movement time ($t_{ballistic}$)", does not equal the length of t_r. Lin et al. (2009) hypothesized that Gan & Hoffmann's (1988) model, shown as Equation 2, could be utilized to predict $t_{ballistic}$.

$$t_{ballistic} = a + b \times \sqrt{d_u} \tag{2}$$

where a and b are experimentally determined constants.

In line with Meyer and his colleagues' motor variability concept, Lin et al. (2009) stated that the total *MT* is affected by ballistic movement variability. However, instead of the impulse-variability model, Lin et al. (2009) hypothesized that Howarth, Beggs & Bowden's, (1971) model, shown as Equation 3, could predict ballistic movement variability.

$$\sigma^2 = a + b \times d_u^2 \tag{3}$$

where σ is the standard deviation of the endpoint distribution measured in the movement direction; a and b are experimentally determined constants.

The last motor property is called the "moving behavior and strategy" that describes how a movement is composed of ballistic movements. While performing a self-paced aiming movement (i.e., Fitts'-type movement), the moving behavior and strategy can be explained by Figure 1.

As shown in Figure 1a, the aiming movement begins with the first ballistic movement that was assumed to move with d_u equal to the movement amplitude. Endpoints of the first ballistic movement as well as the others were determined by the ballistic movement variability model (i.e., Eq. 3). If the first ballistic movement's endpoints are inside the target [Region 1 in Fig. 1a], the movement ends with the first ballistic movement. If the endpoints are in Region 2 [Fig. 1b], two ballistic movements

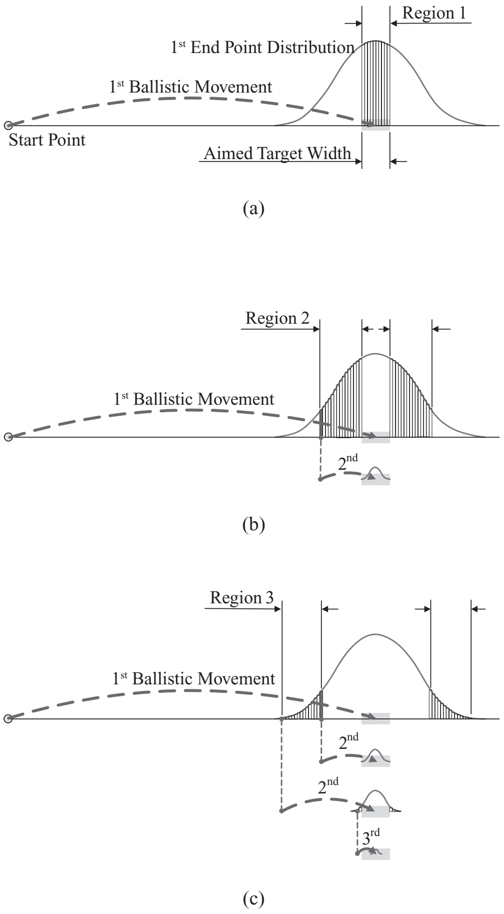

Figure 1. Moving behavior and strategy of the self-paced aiming movements (Lin et al. 2009).

are required to finish the movement. Note that the Region 2 is defined such that all the ballistic movements that start from this region can end inside the target region. And if the endpoints are in Region 3 [Fig. 1c], the movement needs either two or three ballistic movements to finish. Based on this simplified concept, we know that the endpoint distribution magnitudes and the target width together would determine the number of ballistic movements ($n_{ballistic}$) required for completing the aiming movement; the expected $n_{ballistic}$ can be obtained by multiplying every possible combination of ballistic movements for finishing the aiming movement with their associated probabilities. Furthermore, the expected total MT can be obtained by taking $t_{ballistic}$ and t_r into account. Lin et al. (2009) assumed that $t_{ballistic}$ could be predicted by Equation 2 and t_r has a reasonable range from 190 to 290 ms. They also postulated that if a ballistic movement is not the last one to finish the aiming movement and its $t_{ballistic}$ is shorter than t_r, there is a "compensatory delay" of $t_r – t_{ballistic}$ added to that ballistic movement, resulting in the same t_r as Crossman & Goodeve (1963/1983) proposed. Contrary to Crossman & Goodeve's (1963/1983) concept, however, $t_{ballistic}$ can be longer than t_r, which occurs mainly for the first ballistic movements. Furthermore, Lin et al. (2009) asked one research question: whether or not there is a "reaction delay" of $t_r/2$ between the first and the second ballistic movements, indicating the average time required to wait for the next available ballistic movement.

The general model with the moving behavior and strategy introduced above, called the "the self-paced aiming movement model", was only tested against published data in Lin et al. (2009). Due to data limitations, Lin et al. (2009) only demonstrated that the general model can predict the linear speed-accuracy tradeoffs relationship described by Fitts' law.

To further validate this general model, three experiments were designed and tested in this study, comprising (1) the experiment of ballistic movement time and variability, (2) the experiment of normal aiming movement, and (3) the experiment of ballistic aiming movement. The first experiment was designed to measure each individuals' ballistic movement time and variability and to further validate the applications of Gan & Hoffmann's (1988) model and Howarth, et al.'s (1971) model. Due to space limitations, the details of the first experiment will be discussed elsewhere. In this article, the two measured motor properties and the reasonable range of t_r from 190 to 290 ms were treated as inputs of the simulated model programmed based on our self-paced aiming movement model. The outputs of the simulated model were used to predict of the individual participants' actual performance while conducting the two types of aiming movements measured in the last two experiments.

2 METHOD

2.1 *Participants and apparatus*

Two male and two female graduate students, aged from 25–30 years, participated in this pilot study. All the participants were right-handed with normal or corrected-to-normal vision.

A Personal Computer (PC) with a 17" (432 mm) LCD monitor of 1280 × 1024 pixels resolution and an Intous3 305 × 483 mm drawing tablet was used. The PC ran Visual Basic (VB) using three experimental programs that displayed experimental tasks and measured task performance. The drawing tablet was utilized as the input device through all the three experiments. The movement distance ratio between the tablet and the screen was set as 1:1, ensuring equal visual and physical movement distances on the screen and the tablet.

2.2 *Experimental setup and procedures*

While conducting the three experiments the participants sat alongside a dual surface adjustable table on which the monitor and the tablet were placed on the rear and the front surfaces, respectively. To eliminate undesired sources of movement variation other than motor system noise, three strategies were applied. Firstly, while performing movements, the participants wore a nylon half-finger glove and lightly rested their hands on the tablet surface to keep the friction between moving hand and the tablet surface small and constant. Secondly, they were asked to move the stylus tip by moving their whole forearm and by avoiding extending/contracting fingers or wrists to make sure that the measured motor variability was generated from the same sources. Finally, a cardboard screen was placed between their eyes and the tablet to hide the visual feedback from their moving hands so that the only feedback was from the monitor screen.

After informed consent procedures, the participants conducted the three experiments in the following order: (1) the experiment of ballistic movement time and variability, (2) the experiment of ballistic aiming movement, and (3) the experiment of normal aiming movement. Each experiment started with a one-hour practice followed by one formal measurement lasting from 20 to 60 minutes. The participants individually completed all the experiments across three or four appointments within three days. The measured data of ballistic movement time and variability and the two validation experiments are presented in turn.

2.3 *Experiment of ballistic movement time and variability*

This experiment was designed to measure the participants' two motor properties: ballistic movement time and ballistic movement variability. As mentioned above, only the results are presented here. Table 1 shows the participants' ballistic movement time and Table 2 shows ballistic movement variability. Note that instead of Equation 3, it was found that Equation 4, which utilizes ballistic movement distance (d_u) as the predictor, can better predict the ballistic movement variability measured in the movement direction.

$$\sigma^2 = a + b \times d_u \tag{4}$$

Table 1. The measured ballistic movement times represented with Gan & Hoffmann's (1988) model.

Participant	Intercept (ms)	Slope (ms pixel2)	r^2
All	58.32	17.38	0.981
1	27.27	23.16	0.992
2	73.58	16.34	0.952
3	60.99	15.87	0.966
4	71.44	14.15	0.962

Table 2. The measured ballistic movement variability represented with equation 4.

Participant	Intercept (pixel2)	Slope (pixel)	r^2
All	−59.44	2.981	0.985
1	−113.3	3.649	0.899
2	15.22	1.684	0.819
3	−106.7	2.546	0.902
4	−79.28	4.022	0.918

2.4 *Experiment of normal aiming movement*

The purpose of this experiment was to measure the participants' speed-accuracy tradeoffs relationships while performing Fitts-type movements. The measured results were treated as "ground truth" for validating the self-paced aiming movement model. As shown in Figure 2 below, this experiment required the participants to draw lines horizontally from a start point to end within a target line. The independent variables were six *ID*s (2, 2.5, 3, 3.5, 4, and 4.5 bits) and four start point locations, used to diminish the learning of kinesthetic feedback. Each *ID* value included four combinations of target width (*W*) and movement amplitude (*A*). The four values of *W* were 8, 16, 24, and 32 pixels (1 pixel ≅ 0.266 mm), while the values of *A* were determined by Fitts' law. All experimental combinations were replicated 12 times, resulting in a total of 288 trials.

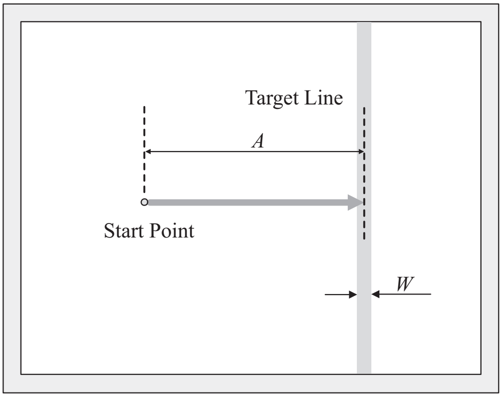

Figure 2. The movement tasks in the experiment of normal aiming movement shown on a monitor screen.

2.5 *Experiment of ballistic aiming movement*

This experiment was designed to measure $n_{ballistic}$ for completing aiming movements according to different *ID* values. Since the self-paced aiming movement model predicts the total *MT* based on the $n_{ballistic}$, the measured $n_{ballistic}$ could be utilized to validate the moving behavior and strategy shown in Figure 1. The Fitts-type movements in this experiment were designed to be performed ballistically; an aiming movement was completed by performing sequential ballistic movements. The task started by pressing down on the pen cursor on the start point. Once the cursor was moved away from the start point toward the target, the visual information disappeared and only reappeared when the ballistic movement stopped. If the endpoint of the ballistic movement was outside the target line, the participants continuously performed ballistic movements from the previous endpoints until the target region was reached. Except for the ballistic movement feature, all the other experimental details were as the same as those in the experiment of normal aiming movement.

3 RESULTS

3.1 *The experiment of normal aiming movement*

The means of *MT* were regressed on to *ID* to give the slopes and intercepts shown in Table 3. Fitts' law predicted both the overall and individual participants' *MT* data very well; it accounted for 98.6% variance of the overall participants' data and at least 95.1% variance of the individual participants' data.

Table 3. Regressions of *MT* on to *ID*.

Participant	Intercept (ms)	Slope (ms/bit)	r^2
All	–71.39	111.1	0.986
1	–181.1	156.6	0.988
2	–66.68	97.96	0.955
3	0.81	90.68	0.951
4	–38.62	99.15	0.974

Table 4. Regressions of $n_{ballistic}$ on to *ID*.

Participant	Intercept (time)	Slope (time/bit)	r^2
All	0.7247	0.2063	0.978
1	0.5369	0.2643	0.917
2	0.7198	0.2155	0.977
3	0.8125	0.1667	0.872
4	0.8294	0.1786	0.907

3.2 *The experiment of ballistic aiming movement*

The means of $n_{ballistic}$ were regressed on to *ID* to give the slopes and intercepts. As shown in Table 4, Fitts' law also predicted both the overall and individual participants' $n_{ballistic}$ data very well; it accounted for 97.8% variance of the overall participants' data and at least 87.2% variance of the individual participants' data.

3.3 *Model testing*

To test the self-paced aiming movement model, the measured motor properties shown in Tables 1 and 2, the t_r values of 190 and 290 ms, and the reaction delay of 0 and $t_r/2$ were treated as inputs of the simulated model. The outputs of the simulation were the predicted *MT* and $n_{ballistic}$ corresponding to the *ID* values measured in the two aiming movement experiments. The simulated model predicted well the linear relationships between $n_{ballistic}$ and *ID* as well as the linear relationships between $n_{ballistic}$ and *ID*. Fitts' law accounted for more than 98.5% and 97.4% variance of the simulated *MT* data and $n_{ballistic}$ data, respectively. Because both the relationships between *MT* and *ID* and the relationships between $n_{ballistic}$ and *ID* can be well accounted for by Fitts' law no matter whether the data were predicted or measured, the validation of the self-paced aiming movement model could be tested by statistically comparing the predicted and the measured linear regression lines.

Tables 5 and 6 show the comparison results of *MT* regression lines when the reaction delay was set as 0 and $t_r/2$, respectively. The highlighted values in the tables indicate no significant difference ($p > 0.05$) between the model predictions and the experimental measurements. No matter whether the reaction delay was set as 0 or $t_r/2$, only two out of 10 comparisons shows no significant difference. However, it seems that when the reaction delay was set as $t_r/2$ the model could predict better, since there are more highlighted values in Table 6.

Graphic representations of the comparisons made for all participants' data are shown in Figure 3 below. As shown in the figure, no matter what settings of t_r and the reaction delay, the model predict longer *MT*s than experimental ones.

Table 5. Comparisons of predicted and experimental regression lines of *MT* data when reaction delay = 0.

		Intercept		Slope	
Participant	(ms)	*t*	*p*	*t*	*p*
All	190	4.71	<0.001	–4.49	< 0.001
	290	6.19	<0.001	–4.23	< 0.001
1	190	3.39	0.0008	–3.97	0.0001
	290	3.63	0.0003	–3.80	0.0002
2	190	4.42	<0.001	–2.23	0.0267
	290	5.62	<0.001	–2.32	0.0209
3	190	–1.36	0.1753	–0.16	0.8712
	290	–1.81	0.0719	0.89	0.3726
4	190	2.18	0.0297	–2.06	0.0405
	290	3.01	0.0028	–1.53	0.1273

Table 6. Comparisons of predicted and experimental regression lines of *MT* data when reaction delay = $t_r/2$.

		Intercept		Slope	
Participant	(ms)	*t*	*p*	*t*	*p*
All	190	2.34	0.0195	1.04	0.2979
	290	1.04	0.2967	4.17	<0.001
1	190	2.07	0.0395	–1.38	0.1675
	290	1.32	0.1868	0.05	0.9606
2	190	3.29	0.0011	0.91	0.3637
	290	2.67	0.0079	2.68	0.0077
3	190	–3.61	0.0004	3.42	0.0007
	290	–4.79	<0.001	5.30	<0.001
4	190	1.08	0.2830	0.86	0.3891
	290	0.41	0.6815	2.68	0.0078

Table 7 shows the comparison results of $n_{ballistic}$ regression lines. The model predict $n_{ballistic}$ better than *MT*. The only significant difference of regression lines was found in Participant 3's data. Graphic representation of comparisons made for

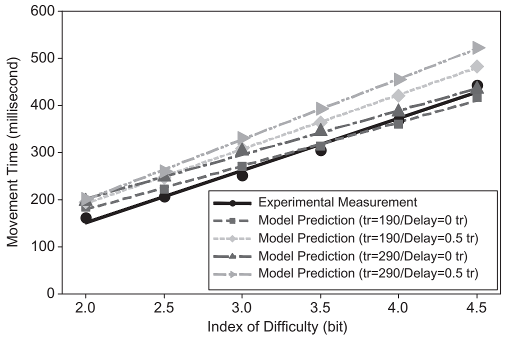

Figure 3. Comparisons of *MT* regression lines made for all participants' data.

Table 7. Regressions of $n_{ballistic}$ on to *ID*.

	Intercept		Slope	
Participant	t	p	t	p
All	–0.7407	0.4591	1.9147	0.0558
1	–0.1665	0.8679	0.4214	0.6737
2	0.4009	0.6888	–0.3021	0.7628
3	–3.5524	0.0004	3.2658	0.0012
4	–0.7007	0.4841	1.6139	0.1076

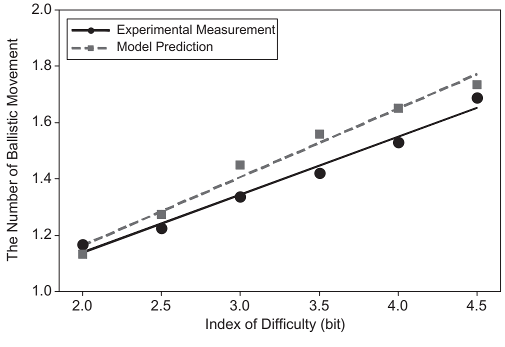

Figure 4. Comparisons of $n_{ballistic}$ regression lines made for all participants' data.

all participants' data is shown in Figure 4 above. Although there is no significant difference between the two regression lines, the model tends to predict more $n_{ballistic}$ than found experimentally.

4 DISCUSSION

The three experiments in this study were feasible for testing the self-paced aiming movement model proposed by Lin et al. (2009). Although detailed contents of the experiment of ballistic movement time and variability are not presented in this article, the experiment successfully captured the participants' two motor properties and demonstrated that they can be described by Gan & Hoffmann's model (1988) model and Equation 4, a modification of Howarth, et al.'s (1971) model. The experiment of normal aiming movement also captured the speed-accuracy tradeoffs relationship described by Fitts' law, which again shows the robustness of Fitts' law. Further, the experiment of ballistic aiming movement successfully measured the number of ballistic movements ($n_{ballistic}$) required for completing the Fitts-type movements. Surprisingly, $n_{ballistic}$ was also linearly related to *ID*. Based on strong linear relationships, the self-paced aiming movement model could be tested by statistical comparisons of the model predictions and the experimental measurements. Although the model did not precisely predict the relationships of *MT* and $n_{ballistic}$, the results showed the feasible application of the designed experiments. The comparisons of *MT* and $n_{ballistic}$ relationships showed that the model predicted longer *MT* and more $n_{ballistic}$. The reason could due to any residual learning effect—the *MT* and $n_{ballistic}$ were measured after the two motor properties. Hence, more practice or multiple measurements of the two motor properties before/after the two types of aiming movement experiments are suggested for future research. Of course, more participants should be recruited.

5 CONCLUSION

This pilot study used three experiments for validating the Lin et al. (2009) self-paced aiming movement model developed to model Fitts' law. The motor properties of ballistic movement time and ballistic movement variability measured in the first experiment were utilized as inputs of the model. The statistical comparisons of the model outputs and the experimental measurements obtained in the last two experiments showed that the designed experiments were feasible for further testing. Some modifications of the experiments were suggested for future research.

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